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Applications of the Anuj Integral Transform in Physical Systems

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Abstract
The derivation of analytical solutions to differential equations is paramount for comprehending the dynamic characteristics of intricate physical systems. This research examines the utilization of the Anuj Integral Transform as a mathematical framework for addressing a variety of interdisciplinary challenges, encompassing damped mechanical oscillations, sequential radioactive decay phenomena, and transient behaviors in RLC electrical circuits. The methodology emphasizes the transformation of linear ordinary differential equations into algebraic representations that facilitate the direct integration of initial conditions while simultaneously minimizing intermediate computational intricacies. The results attained align with traditional analytical solutions and Laplace-based methodologies, thereby substantiating the mathematical legitimacy of the proposed approach. Furthermore, the study elucidates that the Anuj transform provides enhanced algebraic simplicity and computational efficacy, particularly in the context of coupled systems and higher-order differential equations. These conclusions imply that the Anuj Integral Transform may function as a dependable analytical instrument for the modeling of stability and transient dynamics in diverse physical systems.

Keywords: Anuj Integral Transform; Electrical Circuits; Integral Transform Methods; Mathematical Physics; Mechanical Vibrations; Ordinary Differential Equations; Radioactive Decay.

1. Introduction

Differential equations represent the foundational mathematical construct utilized for characterizing the dynamics and progression of physical systems. Numerous phenomena within classical mechanics, nuclear physics, and electrical engineering are regulated by linear ordinary differential equations featuring constant coefficients. These equations are instrumental in modeling fundamental processes, including vibratory motion, sequential radioactive decay, and transient electrical responses [1], [2]. Achieving accurate analytical solutions to these equations is imperative for comprehending the stability, precision, and dynamic characteristics of such systems.

Historically, classical integral transforms, notably the Laplace, Fourier, and Sumudu transforms, have been pivotal in addressing differential equations encountered in the physical sciences [3]. Although these transforms demonstrate considerable efficacy, they often necessitate relatively intricate computational methodologies, especially when integrating complex initial

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conditions or addressing higher-order equations. For instance, while the widely used Laplace transform requires meticulous algebraic manipulation and specific scaling factors during the inverse transformation process, alternative tools like the Sumudu transform possess different scaling properties that may complicate direct physical interpretation in multi-parameter systems. As a result, the advancement of alternative integral transforms that provide algebraic simplicity, direct parameter mapping, and computational efficiency has emerged as a vibrant area of scholarly inquiry.

In this regard, the Anuj Integral Transform, as introduced in [4], has surfaced as a promising analytical instrument for solving ordinary differential equations in various physical applications. This transform facilitates the conversion of differential operators into uncomplicated algebraic expressions, thereby permitting the direct incorporation of initial conditions into the solution process. Since its introduction, the transform has garnered substantial attention in interdisciplinary research. For example, it has been effectively employed to model biological systems such as bacterial growth [5] and population dynamics across interdisciplinary sciences [6]. Moreover, the adaptability of the Anuj transform has been substantiated through its extension to intricate mathematical constructs, such as the Bessel functions of the first kind [7], and even **to solving** formidable ordinary differential equations characterized by variable coefficients [8].

Despite these mathematical advancements, a critical review of the literature reveals a distinct gap: the explicit deployment of the Anuj transform in solving coupled or sequential multi-stage physical phenomena such as multi-component radioactive decay chains or interconnected RLC responses remains inadequately explored. While some recent studies have initiated the use of newly developed transforms for structural beams and simple electrical networks [10], or general physics integrations [12], they often overlook the comparative algebraic reduction achieved when handling high-order initial-value problems simultaneously. This constitutes a significant scientific challenge, as conventional methods frequently become cumbersome when tracking the immediate physical transitions of consecutive states.

The current study seeks to methodically explore and expand the application of the Anuj Integral Transform to specific physical systems, with a particular emphasis on mechanical oscillations [9], nuclear decay chains, and RLC electrical circuits [11]. The primary novelty of this work lies in establishing the explicit algebraic superiority and straightforward inverse formulation of the Anuj transform over traditional frameworks when solving these physical models. By elucidating clear physical interpretations of the findings, this study underscores the potential of the Anuj transform as a dependable, computationally less demanding, and cohesive mathematical instrument for analyzing dynamic behavior and stability within complex physical systems.

2. Methodology: The Anuj Transform

Prior to embarking on the analytical derivations, it is imperative to delineate the formal mathematical framework underpinning the Anuj Integral Transform. This preliminary step establishes the requisite foundation for the conversion of differential equations into algebraic forms that are amenable to analytical scrutiny.

The Anuj Transform is characterized as an integral operator that effectuates a mapping of a function from the temporal domain into an auxiliary transform parameter. Such a mapping permits the articulation of differential relationships as algebraic equations, thus promoting a more systematic and efficient approach to analytical manipulation. The efficacy of this transform has been particularly demonstrated in its capacity to directly incorporate initial conditions into the solution process, thereby setting it apart from various traditional integral transforms [4].

If $g(t)$ is a real function, $t \geq 0$, then the Anuj transform of $g(t)$ is defined by the integral equation (Kumar et al. 2021):

$$\Lambda[g(t)] = r^2 \int_0^\infty g(t) e^{\frac{-t}{r}} dt = G(r), \qquad r > 0 \qquad (1)$$

Where; Λ is called the Anuj transform operator and r is a real parameter.

If $\Lambda[g(t)] = G(r)$, then $g(t)$ is called the inverse Anuj transform of $G(r)$ and mathematically it is denoted by:

$g(t) = \Lambda^{-1}[G(r)]$

where the operator Λ^{-1} is the inverse Anuj transform.

2.1. Anuj Transform and Inverse Anuj Transform of Elementary Functions

In order to implement this transformation proficiently, it is imperative to establish a compendium of standardized transformations pertaining to fundamental functions. This segment provides a reference table encompassing the most prevalent mathematical functions that are typically encountered within the realms of engineering and physics.

The subsequent Anuj transform and inverse Anuj transform for several elementary functions are encapsulated in Table 1 below [4]. Moreover, these equations are derived from the aforementioned reference.

Table (1): Anuj transform of some basic functions

No.	$g(t)$	$\Lambda[g(t)] = G(r)$	$G(r)$	$g(t) = \Lambda^{-1}G(r)$
1	1	r^3	r^3	1
2	t	r^4	r^4	t
3	t^2	$2!\, r^5$	r^5	$\frac{t^2}{2!}$
4	$t^n, n \in N$	$n!\, r^{n+3}$	$r^n, n \in N$	$\frac{t^{n-3}}{(n-3)!}$
5	$t^n, n > -1$	$\Gamma(n+1) r^{n+3}$	$r^{n+3}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6	e^{at}	$\frac{r^3}{1-ar}$	$\frac{r^3}{1-ar}$	e^{at}
7	$\sin(\alpha t)$	$\frac{ar^4}{1+\alpha^2 r^2}$	$\frac{ar^4}{1+\alpha^2 r^2}$	$\sin(\alpha t)$
8	$\cos(\alpha t)$	$\frac{r^3}{1+\alpha^2 r^2}$	$\frac{r^3}{1+\alpha^2 r^2}$	$\cos(\alpha t)$
9	$\sinh(\alpha t)$	$\frac{ar^4}{1-\alpha^2 r^2}$	$\frac{ar^4}{1-\alpha^2 r^2}$	$\sinh(\alpha t)$
10	$\cosh(\alpha t)$	$\frac{r^3}{1-\alpha^2 r^2}$	$\frac{r^3}{1-\alpha^2 r^2}$	$\cosh(\alpha t)$
11	$e^{at} g(t)$	$(1-ar)^2 G\left(\frac{r}{1-ar}\right)$	$(1-ar)^2 G\left(\frac{r}{1-ar}\right)$	$e^{at} g(t)$

2.2. Anuj Transform of Derivatives

The inherent strength of any integral transform resides in its capacity to transmute differentiation into multiplication. This section delineates the operational characteristics that enable the conversion of differential equations into straightforward algebraic equations [4].

These characteristics are employed to resolve ordinary differential equations:

If $\Lambda[g(t)] = G(r)$, then:

i. $\Lambda[g'(t)] = \frac{1}{r}G(r) - r^2 g(0)$

ii. $\Lambda[g''(t)] = \frac{1}{r^2}G(r) - rg(0) - r^2\frac{d}{dr}g(0)$

iii. $\Lambda[g'''(t)] = \frac{1}{r^3}G(r) - g(0) - r\frac{d}{dr}g(0) - r^2\frac{d^2}{dr^2}g(0)$

2.3. Comparison with Classical Integral Transforms

The Anuj Integral Transform exhibits conceptual parallels with the classical Laplace transform, as both methodologies transmute differential equations into algebraic expressions within the transform domain [4]. This transformation facilitates the analytical resolution of initial value problems that are frequently encountered in physical systems.

Nonetheless, the Anuj transform integrates initial conditions directly into the transformed expressions in a manner that may diminish intermediate computational steps and streamline algebraic manipulation in specific applications. Despite the variances in formulation, the analytical solutions yielded through the Anuj transform are congruent with those derived using the Laplace transform, thereby substantiating the mathematical validity and dependability of the method.

The subsequent table encapsulates a comparative analysis between the Laplace and Anuj transforms for derivatives.

Table 2: Comparison between Laplace and Anuj transforms for Derivatives

Feature	Laplace Transform (L)	Anuj Transform ($\boldsymbol{\Lambda}$)
First Derivative	$s\,L[f(t)] - f(0)$	$\frac{1}{r}\,\Lambda[g(t)] - r^2 g(0)$
Second Derivative	$s^2 L[f(t)] - s\,f(0) - f'(0)$	$\frac{1}{r^2}\,\Lambda[g(t)] - rg(0) - r^2\,g'(0)$
Treatment of Initial Conditions	Appears explicitly in algebraic form	Incorporated directly within transform formulation
Computational Effort	Standard procedural steps	Reduced intermediate steps in many cases
Algebraic Simplicity	Depends on problem structure	Highly streamlined for linear ODEs

3. Physical Applications of the Anuj Transform

3.1. Application to Mechanics

In this particular application, we examine the dynamics of a particle subjected to both a restoring force directed towards the origin and a damping force that is proportional to its instantaneous velocity. This model exemplifies real-world mechanical systems that exhibit energy dissipation over time [9]. Adhering to the analytical framework established by the Anuj Transform [10], we consider the following problem:

Problem: A particle P of mass $m = 2\,g$ moves along the x-axis, subjected to a restoring force proportional to its displacement from the origin O, given by a force numerically equal to $8X$. The particle is initially released from rest at $X(0) = 10\,cm$. Determine the position $X(t)$ of the particle at any subsequent time t under the following two conditions: *(a) Undamped Motion:* A conservative system with no resistive forces. *(b) Damped Motion:* A system where a damping force numerically equal to 8 times the instantaneous velocity acts on the particle.

Solution:

(a): Undamped Motion (Simple Harmonic Motion). The system is conservative, and no external resistive forces act on the particle. The governing equation is:

$$2\frac{d^2X}{dt^2} = -8X \quad \Rightarrow \quad \frac{d^2X}{dt^2} + 4X = 0 \qquad (2)$$

with the initial conditions $X(0) = 10,\ X'(0) = 0$

Applying the Anuj transform to both sides of Eq. (2), we have:

$$\Lambda\left[\frac{d^2X}{dt^2}\right] + 4\Lambda[X] = \Lambda[0]$$

By using the Anuj transform of derivatives, we obtain:

$$\frac{1}{r^2}\Lambda(X) - rX(0) - r^2\frac{d}{dr}X(0) + 4\Lambda(X) = 0$$

Now, by using given initial conditions $X(0) = 10,\ X'(0) = 0$ and simplifying the above expression, we obtain:

$$\Lambda(X(t)) = \frac{10r^3}{1+4r^2}$$

Applying inverse Anuj transform, we get:

$$X(t) = \Lambda^{-1}\left[\frac{10r^3}{1+4r^2}\right] = 10\Lambda^{-1}\left[\frac{r^3}{1+4r^2}\right] = 10\cos(2t)$$

(b). Damped Motion: In this case, a damping force numerically equal to 8 times the instantaneous velocity $(8X')$ acts on the particle. The motion is governed by the following second-order differential equation:

$$2\frac{d^2X}{dt^2} = -8X - 8\frac{dX}{dt}$$

$$\Rightarrow \quad \frac{d^2X}{dt^2} + 4\frac{dX}{dt} + 4X = 0 \qquad (3)$$

With the initial conditions $X(0) = 10,\ X'(0) = 0$

Applying the Anuj transform to both sides of Eq. (3), we have:

$$\Lambda\left[\frac{d^2X}{dt^2}\right] + 4\Lambda\left[\frac{dX}{dt}\right] + 4\Lambda[X] = \Lambda[0]$$

By using the Anuj transform of derivatives and apply the above initial conditions.

$$\frac{1}{r^2}\Lambda(X) - rX(0) - r^2\frac{d}{dr}X(0) + \frac{4}{r}\Lambda(X) - 4r^2X(0) + 4\Lambda(X) = 0$$

$$\Lambda(X)\left[\frac{1}{r^2}+\frac{4}{r}+4\right]=10r+40r^2$$

$$\Lambda(X)\left[\frac{1+4r+4r^2}{r^2}\right]=10r+40r^2$$

$$\Lambda(X)=\frac{10r^3+40r^4}{1+4r+4r^2}=\frac{10r^3+20r^4+20r^4}{(1+2r)^2}=\frac{10r^3(1+2r)+20r^4}{(1+2r)^2}$$

$$\Lambda(X)=\frac{10r^3}{(1+2r)}+\frac{20r^4}{(1+2r)^2}$$

Now, applying inverse Anuj transform, we get:

$$X(t)=\Lambda^{-1}\left[\frac{10r^3}{(1+2r)}\right]+\Lambda^{-1}\left[\frac{20r^4}{(1+2r)^2}\right]=10\Lambda^{-1}\left[\frac{r^3}{(1+2r)}\right]+20\Lambda^{-1}\left[\frac{r^4}{(1+2r)^2}\right]$$

$$\Rightarrow \quad X(t)=10e^{-2t}+20te^{-2t}$$

Now, by applying the inverse Anuj transform and utilizing the shifting property specified in Table 1 (No. 11), the explicit time-domain solution is obtained. The resolution elucidates the critically damped motion of the particle. From a physical perspective, this result signifies that the damping force is impeccably calibrated to revert the particle to its equilibrium position at the origin O as swiftly as possible without oscillation. Such a model functions as a fundamental element in mechanical and structural design, ensuring stability and safety in systems such as automotive shock absorbers and vibrating structures [3].

3.2. Application to Electrical Circuits

Series RLC circuits represent ideal models for the examination of transient responses in dynamic systems. In accordance with Kirchhoff's Voltage Law, the aggregate of the potential drops across the circuit elements is equivalent to the applied electromotive force [2], [11]. In this section, we employ the Anuj Transform to determine the electric charge, showcasing its efficacy in addressing circuits characterized by constant parameters and initial conditions [10].

Problem: A series circuit consists of an inductor $L=2\,H$, a resistor $R=16\Omega$, and a capacitor $C=0.02\,F$, connected to an electromotive force (*e.m.f*) $E=300\,V$, at $T=0$. If the initial charge on the capacitor and the current in the circuit are both zero $(Q(0)=0, I(0)=0)$, find the charge $Q(T)$ and the current $I(T)$ at any time $T>\ 0$.

Solution:

By Kirchhoff's Voltage Law, the governing differential equation is:

$$L\frac{d^2Q}{dT^2}+R\frac{dQ}{dT}+\frac{Q}{C}=E$$

Substituting the given values $(L=2, R=16, C=0.02, E=300)$:

$$2\frac{d^2Q}{dT^2}+16\frac{dQ}{dT}+\frac{Q}{0.02}=300$$

$$\frac{d^2Q}{dT^2}+8\frac{dQ}{dT}+25Q=150 \qquad (4)$$

By applying the Anuj transform to Eq. (4), we have:

$$\Lambda\left[\frac{d^2Q}{dT^2}\right] + 8\Lambda\left[\frac{dQ}{dT}\right] + 25\Lambda(Q) = \Lambda(150)$$

$$\frac{1}{r^2}\Lambda(Q(T)) - rQ(0) - r^2\frac{d}{dr}Q(0) + 8\left[\frac{1}{r}\Lambda(Q(T)) - r^2Q(0)\right] + 25\Lambda(Q(T)) = 150r^3$$

With initial conditions $Q(0) = 0, I(0) = \frac{d}{dT}Q(0) = 0$.

$$\Lambda(Q(T))\left[\frac{1}{r^2} + \frac{8}{r} + 25\right] = 150r^3$$

$$\Lambda(Q(T))\left[\frac{1 + 8r + 25r^2}{r^2}\right] = 150r^3$$

$$\Lambda(Q(T)) = \frac{150r^5}{1 + 8r + 25r^2}$$

$$= \frac{6r^3 + 48r^4 + 150r^5 - 6r^3 - 24r^4 - 24r^4}{1 + 8r + 25r^2}$$

$$= \frac{6r^3(1 + 8r + 25r^2) - 6r^3(1 + 4r) - 24r^4}{1 + 8r + 25r^2}$$

$$\Lambda(Q(T)) = 6r^3 - \frac{6r^3(1 + 4r)}{(1 + 4r)^2 + 9r^2} - \frac{24r^4}{(1 + 4r)^2 + 9r^2}$$

Now, applying the inverse Anuj transform and after some algebraic steps, we obtain:

$$Q(T) = \Lambda^{-1}[6r^3] - \Lambda^{-1}\left[\frac{6r^3(1 + 4r)}{(1 + 4r)^2 + 9r^2}\right] - \Lambda^{-1}\left[\frac{24r^4}{(1 + 4r)^2 + 9r^2}\right]$$

$$Q(T) = \Lambda^{-1}[6r^3] - 6\Lambda^{-1}\left[\frac{\frac{r^3}{(1 + 4r)}}{1 + 9\left(\frac{r}{1 + 4r}\right)^2}\right] - 24\Lambda^{-1}\left[\frac{\frac{r^4}{(1 + 4r)^2}}{1 + 9\left(\frac{r}{1 + 4r}\right)^2}\right]$$

$$Q(T) = \Lambda^{-1}[6r^3] - 6\,\Lambda^{-1}\left[(1 + 4r)^2\frac{\left[\frac{r}{(1 + 4r)}\right]^3}{1 + 9\left(\frac{r}{1 + 4r}\right)^2}\right] - 8\Lambda^{-1}\left[(1 + 4r)^2\frac{3\left[\frac{r}{(1 + 4r)}\right]^4}{1 + 9\left(\frac{r}{1 + 4r}\right)^2}\right]$$

Now, by applying the inverse Anuj transform and utilizing the shifting property specified in Table 1 (No. 11), we obtain the final time-domain response:

$$\Rightarrow\ Q(T) = 6 - 6e^{-4T}\cos(3T) - 8e^{-4T}\sin(3T)$$

The current $I(T)$ is obtained by differentiating the charge with respect to time

$$I(T) = \frac{dQ}{dT} = 50e^{-4T}\sin(3T)$$

The findings imply an underdamped oscillatory response, characterized by the charge and current oscillating at a damped angular frequency while concurrently exhibiting an exponential decay as a consequence of the resistance R. Ultimately, the system achieves a steady state in which the current becomes null and the charge reaches its maximal value of 6 units [2], [10].

3.3. Application to Nuclear Physics: Successive Radioactive Decay

In nuclear physics, we study decay chains where a parent nucleus decays into a daughter nucleus, which may also be radioactive. The goal is to determine the population of each component over time using the fundamental laws of radioactive transformation [1]. In this section, we apply the Anuj Transform to solve the system of coupled differential equations governing this process, demonstrating its accuracy in nuclear modeling [12].

Problem: In nuclear physics, successive radioactive decay processes are governed by systems of first-order linear differential equations. Consider a parent radioactive nucleus N_1 that decays into a daughter nucleus N_2, where λ_1 and λ_2 represent their respective decay constants. Determine the amount of $N_1(\boldsymbol{t})$ and $N_2(\boldsymbol{t})$ at any time t, assuming that at $t = 0$, only the parent nucleus is present $N_1(0) = N_0$ and $N_2(0) = 0$.

Solution

Assuming the decay process follows the Bateman model, the governing equations are:

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \; and \; \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

With initial condition: $N_1(0) = N_0$, $N_2(0) = 0$

$$\frac{dN_1}{dt} + \lambda_1 N_1 = 0 \qquad (5)$$

Applying the Anuj transform to Eq. (5), we obtain:

$$\Lambda\left[\frac{dN_1}{dt}\right] + \lambda_1 \Lambda(N_1) = 0$$

By using the Anuj transform of derivatives, we obtain:

$$\frac{1}{r}\Lambda(N_1) - r^2 N_1(0) + \lambda_1 \Lambda(N_1) = 0$$

$$\Lambda(N_1)\left(\frac{1}{r} + \lambda_1\right) = r^2 N_1(0)$$

$$\Lambda(N_1)\left(\frac{1 + \lambda_1 r}{r}\right) = r^2 N_1(0)$$

$$\Lambda(N_1) = N_1(0)\frac{r^3}{1 + \lambda_1 r}$$

Now, by using given first initial condition: $N_1(0) = N_0$

$$\Lambda(N_1) = N_0 \frac{r^3}{1 + r\lambda_1} \qquad (6)$$

Applying the inverse Anuj transform to both sides, we yield:

$$N_1(t) = N_0 \Lambda^{-1}\left[\frac{r^3}{1 + \lambda_1 r}\right]$$

$$N_1(\boldsymbol{t}) = N_0 \boldsymbol{e}^{-\lambda_1 \boldsymbol{t}}$$

$$\because \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \Rightarrow \frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_1 \qquad (7)$$

Applying the Anuj transform to Eq. (7), we obtain:

$$\Lambda\left[\frac{dN_2}{dt}\right] + \lambda_2 \Lambda(N_2) = \lambda_1 \Lambda(N_1)$$

Applying the Anuj transform of derivatives to the governing equation, we obtain:

$$\frac{1}{r}\Lambda(N_2) - r^2 N_2(0) + \lambda_2 \Lambda(N_2) = \lambda_1 \Lambda(N_1)$$

$$\Lambda(N_2)\left(\frac{1}{r} + \lambda_2\right) = \lambda_1 \Lambda(N_1)$$

$$\Lambda(N_2)\left(\frac{1+\lambda_2 r}{r}\right) = \lambda_1 \Lambda(N_1)$$

$$\Lambda(N_2) = \lambda_1 \Lambda(N_1)\left(\frac{r}{1+\lambda_2 r}\right) \qquad (8)$$

Since $N_2(0) = 0$

Substituting Eq. (6) into Eq. (8), we obtain:

$$\Lambda(N_2) = \lambda_1 N_0 \left(\frac{r^3}{1+\lambda_1 r}\right)\left(\frac{r}{1+\lambda_2 r}\right)$$

$$\Lambda(N_2) = \lambda_1 N_0 \left(\frac{r^4}{(1+\lambda_1 r)(1+\lambda_2 r)}\right)$$

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(\frac{r^4(\lambda_2 - \lambda_1)}{(1+\lambda_1 r)(1+\lambda_2 r)}\right)$$

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(\frac{\lambda_2 r^4 - \lambda_1 r^4}{(1+\lambda_1 r)(1+\lambda_2 r)}\right)$$

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(\frac{r^3 + \lambda_2 r^4 - r^3 - \lambda_1 r^4}{(1+\lambda_1 r)(1+\lambda_2 r)}\right)$$

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left(\frac{r^3(1+\lambda_2 r) - r^3(1+\lambda_1 r)}{(1+\lambda_1 r)(1+\lambda_2 r)}\right)$$

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left[\frac{r^3}{1+\lambda_1 r} - \frac{r^3}{1+\lambda_2 r}\right]$$

Applying the inverse Anuj transform, we obtain:

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \Lambda^{-1}\left[\frac{r^3}{1+\lambda_1 r} - \frac{r^3}{1+\lambda_2 r}\right]$$

$$\Lambda(N_2) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 \left[\Lambda^{-1}\left(\frac{r^3}{1+\lambda_1 r}\right) - \Lambda^{-1}\left(\frac{r^3}{1+\lambda_2 r}\right)\right]$$

$$N_2(\boldsymbol{t}) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 (\boldsymbol{e}^{-\lambda_1 \boldsymbol{t}} - \boldsymbol{e}^{-\lambda_2 \boldsymbol{t}})$$

This analytical outcome precisely aligns with the classical Bateman equation derived via traditional integration or the Laplace transform [1], [12]. The Anuj transform adeptly decouples the first-order system with minimal algebraic steps, demonstrating its efficacy in monitoring transient physical populations over time.

4. Discussion

The analytical outcomes derived from this investigation substantiate the efficacy of the Anuj Integral Transform as an instrumental mathematical mechanism for the modeling of physical phenomena. The observed congruence between the generated algebraic solutions and established physical principles corroborates the mathematical integrity of the methodology.

One of the principal merits of the Anuj transform, as elucidated in the comparative assessment with the Laplace transform in Table 2, is its capacity to incorporate initial conditions directly into the algebraic formulation. This attribute diminishes intermediate computational procedures, especially by reducing the necessity for the determination of integration constants through distinct methodologies. This observation aligns with the conclusions drawn by Kumar et al. [4] and Rashdi [8], who asserted that the transform facilitates the simplification of differential equation treatment with both constant and variable coefficients.

In the context of mechanical applications, the transform exhibited efficacy in resolving second-order differential equations that govern damped motion. The resultant analytical expression for accurately characterizes the transition from harmonic motion to damped oscillations as a function of the damping coefficient. By directly integrating the initial displacement and velocity conditions, the method circumvents the substitution steps that are typically requisite in conventional solution methodologies.

Likewise, in the modeling of electrical RLC circuits, the Anuj transform rendered a direct representation of the transient response within the transform domain. The derived expression for the charge function elucidates the system's evolution towards steady-state behavior, a crucial consideration in the analysis of circuit stability. Recent investigations by Almardy et al. [10] bolster the assertion that contemporary integral transforms can yield enhanced algebraic convenience in the modeling of time-dependent electrical systems.

Moreover, the application to sequential radioactive decay exemplifies the transform's proficiency in managing coupled first-order differential equations. The analytical expressions for $N_1(t)$and $N_2(t)$ align with classical nuclear physics frameworks delineated by Krane [1], while concurrently exhibiting compatibility with contemporary integration methodologies [12].

The correlation between the Anuj transforms and specialized mathematical functions, including Bessel functions [7], indicates potential applicability that extends beyond the linear systems scrutinized in this investigation. In summary, the findings suggest that the Anuj transform offers a systematic and dependable framework for the analysis of dynamic physical processes while preserving mathematical lucidity.

5. Recommendations and Future Work

In light of the analytical proficiency exhibited by the Anuj Integral Transform in assessing linear and coupled physical systems, the subsequent recommendations are posited for forthcoming inquiries:

1- Extension to Fractional Order Systems: Subsequent investigations should examine the efficacy of the Anuj transform in addressing fractional differential equations, which are of paramount significance in advanced polymer mechanics and anomalous diffusion profiles.
2- Application to Non-Linear Physical Models: It is advised to integrate the Anuj transform with homotopy perturbation methods (HPM) or variational iteration methods (VIM) to address non-linear physical phenomena such as substantial amplitude pendulum oscillations.
3- Numerical Validation: Comparative analyses comparing the analytical outcomes of the Anuj transform with high-order numerical methodologies (e.g., Runge-Kutta 4th order) should be undertaken to establish computational runtime benchmarks.

6. Conclusion

This research elucidates that the Anuj Integral Transform serves as a potent analytical instrument for addressing ordinary differential equations that emerge within physical systems. Through its implementation in domains such as mechanics, nuclear physics, and electrical engineering, the transform has demonstrated its capability to streamline mathematical processes while maintaining the precision of solutions. The congruence of the resultant findings with traditional Laplace-based solutions further substantiates the mathematical robustness of the methodology. The physical interpretations derived from the results, encompassing damping dynamics and trends in radioactive activity, substantiate the transform's credibility. These revelations contribute to the simplification of analytical processes in applied physics and engineering, particularly in contexts demanding swift evaluations of stability.

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